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SIMILARITY CRITERIA OF FLOWS THAT CHARACTERIZE THE THERMODYNAMIC PROPERTIES OF LIQUIDS AND REAL GASES

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The authors obtained the criteria of hydrodynamic and thermal similarity of flows that characterize the thermodynamic properties of liquids and real gases. A mathematical model of processes in liquids and real gases is developed in dimensionless form.

The known criteria of hydrodynamic and thermal similarity of gas flows are found by analysis of the equations of the dynamics of flow of a viscous ideal gas with constant heat capacity that are written in dimensionless form [1, 2]. For the similarity of flows of real gases and liquids to be ensured, in addition to the geometric and kinematic similarity and the equality of the known criteria, the conditions of similarity of the molecular structure and states of substances whose motions are compared (conditions of thermodynamic similarity) must hold [3, 4]. With thermodynamic similarity the compared flows will be similar if liquids at identical points and at identical instants of time are in the corresponding (i.e., thermodynamically equivalent) states.

1. In engineering, if becomes necessary to use, in modeling of flows, real gases and liquids which are not thermodynamically similar to full-scale ones. For determination of the similarity criteria of real gases and liquids which characterize their thermodynamic properties, it is necessary to supplement the system of differential dynamic equations by the thermal equation of state [5]

$$d\rho = (\partial\rho/\partial T)_p\, dT + (\partial\rho/\partial p)_T\, dp \tag{1}$$

and the caloric equation of state [5]

$$dh = (\partial h/\partial T)_p\, dT + (\partial h/\partial p)_T\, dp\ . \tag{2}$$

We replace the dimensional partial derivatives in Eqs. (1) and (2) by the following dimensionless derivatives [5]: the relative isobaric coefficient of volumetric expansion $\alpha T = \alpha/\alpha_0 = (T/v)(\partial v/\partial T)_p$, the Grüneisen criterion $\Gamma = \alpha a^2/c_p = (c_p/c_v - 1)/(\alpha T) = (\rho/T)(\partial T/\partial\rho)_s$, and the adiabatic index $\mathrm{k} = \rho a^2/p = mc_p/c_v = K_s/p = (\rho/p)(\partial p/\partial\rho)_s$. We obtain

$$d\rho/\rho = -\,\alpha T dT/T + (\alpha T\Gamma/\mathrm{k})\, dp/p\ ,\quad dh/(c_pT) = dT/T + [(1-\alpha T)\,\Gamma/(\mathrm{k}\alpha T)]\, dp/p\ . \tag{3}$$

In an ideal gas with constant heat capacity, the adiabatic index is equal to the ratio of the heat capacities $\mathrm{k} = \gamma = c_p/c_v = \text{const}$, $\Gamma = \gamma - 1$. In liquids and dense gases (when $\rho/\rho_{\text{cr.p}} > 1.6$), where the adiabatic elasticity moduli change weakly, at s = const and T = const, the index k is inversely proportional to pressure and cannot be taken to be constant.

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In the general case, the relative change in the velocity of sound can be represented in the form

$$da/a = 0.5\,(dK_s/K_s - d\rho/\rho) = 0.5\left\{[(\kappa + \psi)/\mathrm{k}]\,dp/p - (\psi + 1)\,d\rho/\rho\right\}. \tag{4}$$

Here

$$\kappa = \mathrm{k} + p\,(\partial \mathrm{k}/\partial p)_s = (\partial K_s/\partial p)_s\,; \quad \psi = (v/K_s)\,(\partial K_s/\partial v)_p = [1/(\alpha K_s)]\,(\partial K_s/\partial T)_p\,. \tag{5}$$

If $(\partial K_s/\partial p)_s$ = const, then $\kappa = (\mathrm{k}_2 p_2 - \mathrm{k}_1 p_1)/(p_2 - p_1)$.

In contrast to the adiabatic index, the parameter $B_s = p(\mathrm{k} - \kappa)/\kappa$ and the dimensionless combinations κ and ψ in liquids and dense gases change weakly or remain constant with substantial changes in the pressure and temperature. In calculation and modeling of flows of liquids and dense gases, the parameters B_s, κ, and ψ can often be assumed to be constant and equal to their mean values in the considered processes.

When expressions (3)–(5) are used, the equations of inversion of the effects [6, 7] for changes in the velocity, density, pressure, temperature, and Mach number M = w/a in a one-dimensional flow of a liquid take the form

$$
\begin{gathered}
(\mathrm{M}^2 - 1)\,dw/w = dF/F - dL_{\mathrm{eng}}/a^2 - (\Gamma + 1)\,dL_{\mathrm{fr}}/a^2 - \Gamma dQ/a^2\,,\\
(\mathrm{M}^2 - 1)\,d\rho/\rho = -\,\mathrm{M}^2\,dF/F + dL_{\mathrm{eng}}/a^2 + (\Gamma + 1)\,dL_{\mathrm{fr}}/a^2 + \Gamma dQ/a^2\,,\\
(\mathrm{M}^2 - 1)\,dp/p = \mathrm{k}\,[-\,\mathrm{M}^2\,dF/F + dL_{\mathrm{eng}}/a^2 + (\Gamma\mathrm{M}^2 + 1)\,dL_{\mathrm{fr}}/a^2 + \Gamma\mathrm{M}^2 dQ/a^2]\,,\\
(\mathrm{M}^2 - 1)\,dT/T = \Gamma\left\{-\,\mathrm{M}^2\,dF/F + dL_{\mathrm{eng}}/a^2 + [\Gamma\mathrm{M}^2 + 1 + (\mathrm{M}^2 - 1)/(\alpha T)]\,dL_{\mathrm{fr}}/a^2 +\right.\\
\left. + [\Gamma\mathrm{M}^2 + (\mathrm{M}^2 - 1)/(\alpha T)]\,dQ/a^2\right\},\\
(\mathrm{M}^2 - 1)\,d\mathrm{M}^2/\mathrm{M}^2 = (\mathrm{M}^2 - 1)\,dw^2/w^2 - (\mathrm{M}^2 - 1)\,da^2/a^2 = [2 + (\kappa - 1)\,\mathrm{M}^2]\,dF/F -\\
-\,(\kappa + 1)\,dL_{\mathrm{eng}}/a^2 - \left\{\kappa + 1 + \Gamma\,[2 + (\kappa - 1)\,\mathrm{M}^2 + (\psi + 1)\,(\mathrm{M}^2 - 1)]\right\}dL_{\mathrm{fr}}/a^2 -\\
-\,\Gamma\,[2 + (\kappa - 1)\,\mathrm{M}^2 + (\psi + 1)\,(\mathrm{M}^2 - 1)]\,dQ/a^2\,.
\end{gathered} \tag{6}
$$

The known formula [6] for changes in the Mach number in a one-dimensional flow of an ideal gas

$$
\begin{gathered}
(\mathrm{M}^2 - 1)\,d\mathrm{M}^2/\mathrm{M}^2 = [2 + (\gamma - 1)\,\mathrm{M}^2]\,dF/F - (\gamma + 1)\,dL_{\mathrm{eng}}/a^2 - \gamma\,[2 + (\gamma - 1)\,\mathrm{M}^2]\,dL_{\mathrm{fr}}/a^2 -\\
-\,(\gamma - 1)\,(1 + \gamma\mathrm{M}^2)\,dQ/a^2
\end{gathered} \tag{7}
$$

is a particular case of Eq. (6) for $\kappa = \mathrm{k} = \gamma$, $\Gamma = \gamma - 1$, $\alpha T = 1$, and $\psi = 0$.

In an isentropic process of a liquid or a dense gas for B_s = const, κ = const, and Γ = const, from (6) we obtain [8, 9] the equations of isentropy and the thermodynamic functions of the number M and the reduced velocity $\lambda = w/a_{\mathrm{cr}} = \mathrm{M}\{(\kappa + 1)/[2 + (\kappa - 1)\mathrm{M}^2]\}^{1/2}$:

$$\mathrm{k} = (p + B_s)\,\kappa/p\,, \quad a^2 = \mathrm{k}p/\rho = (p + B_s)\,\kappa/\rho\,; \quad \rho_2/\rho_1 = v_1/v_2 = [(p_2 + B_s)/(p_1 + B_s)]^{1/\kappa}\,;$$

$$T_2/T_1 = (\rho_2/\rho_1)^{\Gamma} = [(p_2 + B_s)/(p_1 + B_s)]^{\Gamma/\kappa}\,; \quad \mathrm{k}pv^{\kappa} = (p + B_s)\,\kappa v^{\kappa} = \mathrm{const}\,;$$

$$\rho^*/\rho = [1 + (\kappa - 1)\, M^2/2]^{1/(\kappa-1)}\,; \quad a^*/a = [1 + (\kappa - 1)\, M^2/2]^{1/2}\,;$$

$$(p^* + B_s)/(p + B_s) = k^* p^*/kp = [1 + (\kappa - 1)\, M^2/2]^{\kappa/(\kappa-1)}\,;$$

$$T^*/T = [1 + (\kappa - 1)\, M^2/2]^{\Gamma/(\kappa - 1)}\,; \quad \rho/\rho^* = [1 - (\kappa - 1)\, \lambda^2/(\kappa + 1)]^{1/(\kappa-1)}\,; \tag{8}$$

$$a/a^* = [1 - (\kappa - 1)\, \lambda^2/(\kappa + 1)]^{1/2}\,; \quad (p + B_s)/(p^* + B_s) = [1 - (\kappa - 1)\, \lambda^2/(\kappa + 1)]^{\kappa/(\kappa-1)}\,;$$

$$T/T^* = [1 - (\kappa - 1)\, \lambda^2/(\kappa + 1)]^{\Gamma/(\kappa-1)}\,.$$

Isentropic specific works of compression and expansion of a liquid and a dense gas are determined from the formulas

$$L_{c.s} = \left\{\kappa\, (p_1 + B_s)/[(\kappa - 1)\, \rho_1]\right\} \left\{[(p_2 + B_s)/(p_1 + B_s)]^{(\kappa-1)/\kappa} - 1\right\};$$

$$L_{\exp.s} = \left\{\kappa\, (p_1 + B_s)/[(\kappa - 1)\, \rho_1]\right\} \left\{1 - [(p_2 + B_s)/(p_1 + B_s)]^{(\kappa-1)/\kappa}\right\}. \tag{9}$$

Particular cases of formulas (8) and (9) for $B_s = 0$, $\kappa = k = \gamma$, and $\Gamma = k - 1$ are the adiabatic Poisson equations, the known gasdynamic functions, and the equations of isentropic works.

From (8) it follows that the hydrodynamic function of the Mach number M and the reduced velocity λ of the isentropic flow of a liquid and a dense gas depend, in contrast to the gasdynamic functions of the flow of an ideal gas, on the values of the parameters κ and Γ and on the value of the ratio $k^*/\kappa = B_s/p^* + 1$ rather than on the ratio of the heat capacities γ. If $B_s = 0$, $\kappa = k$, but Γ is not equal to the difference $k - 1$, we obtain the equation of isentropy and the hydrodynamic functions of the isentropic flow of a real gas, in the considered process of which the criteria k and Γ may be assumed to be constant.

2. Following [10], we consider the conditions of flow of a liquid or a real gas past a full-scale object and its model. The equations of motion and energy determining model flows are written in the parameters of the full-scale flow. All quantities entering these equations are expressed in terms of the fractions of the corresponding quantities for an undisturbed flow at a distance from the body (w_∞, p_∞, ρ_∞). The characteristic values of time and dimensions of the body are denoted as t_0 and l. The excess temperature is expressed in the fractions of the difference of the temperatures of an incoming flow and the body wall: $\Delta T/\Delta T_0 = (T - T_w)/(T_\infty - T_w)$. The dimensionless quantities are denoted by an overbar: $\bar{t} = t/t_0$, $\bar{x} = x/l$, $\bar{w} = w/w_\infty$, $\bar{p} = p/p_\infty$, $\bar{\rho} = \rho/\rho_\infty$, $\bar{\nu} = \nu/\nu_\infty$, $\bar{\lambda} = \lambda/\lambda_\infty$, and $\bar{T} = T/\Delta T_0$. We represent the relative change in the velocity in the form $dw/w = d\mathrm{M}/\mathrm{M} + da/a$, where da/a is determined from formula (4).

We obtain the following equalities:

$$[l\bar{w}/(w_\infty t_0)] \left\{(1/\mathrm{M})\, (\partial \mathrm{M}/\partial \bar{t}\,) + [(\kappa + \psi)/(2k\bar{p})]\, (\partial \bar{p}/\partial \bar{t}\,) - [(1 + \psi)/(2\bar{\rho}\,)]\, (\partial \bar{\rho}/\partial \bar{t}\,)\right\} +$$

$$+ \bar{w}^{\,2} \left\{(1/\mathrm{M})\, (\partial \mathrm{M}/\partial \bar{x}\,) + [(\kappa + \psi)/(2k\bar{p}\,)]\, (\partial \bar{p}/\partial \bar{x}\,)\right\} - [(1 + \psi)/(2\bar{\rho}\,)]\, (\partial \bar{\rho}/\partial \bar{x}\,) = gl/w_\infty^2 -$$

$$- [p_\infty/(\rho_\infty w_\infty^2)]\, (1/\bar{\rho}\,)\, (\partial \bar{p}/\partial \bar{x}\,) + (4\bar{\nu}/3)\, [\nu_\infty/(w_\infty l)]\, (\partial^2 \bar{w}/\partial \bar{x}^{\,2}) + (4\bar{\nu}/3)\, [\nu_\infty/(w_\infty l)]\, (\partial \bar{\nu}/\partial \bar{x}\,)\, (\partial \bar{w}/\partial \bar{x}\,)\,; \tag{10}$$

$$[l/(w_\infty t_0)]\, [w_\infty^2/(c_{p\infty}\, \Delta T_0)]\, (\bar{w}\partial \bar{w}/\partial \bar{t}\,) + [l/(w_\infty t_0)]\, [\alpha T/(\alpha T)_\infty]\, [k\, \Gamma_\infty \bar{p}/(k_\infty \Gamma \bar{\rho})]\, (\partial \bar{T}/\partial \bar{t}) +$$

$$+ [w_\infty^2/(c_{p\infty}\, \Delta T_0)]\, (\bar{w}^2 \partial \bar{w}/\partial \bar{x}\,) + \bar{w}\, [\alpha T/(\alpha T)_\infty]\, [k\, \Gamma_\infty \bar{p}\, \bar{T}_\infty/(k_\infty \Gamma \bar{\rho}\, \bar{T})]\, (\partial \bar{T}/\partial \bar{x}) =$$

$$= [l/(w_\infty t_0)]\,[p_\infty/(\rho_\infty w_\infty^2)]\,[w_\infty^2/(c_{p\infty}\,\Delta T_0)]\,(\alpha T/\bar\rho)\,(\partial\bar p/\partial\bar t) +$$

$$+ [p_\infty/(\rho_\infty w_\infty^2)]\,[w_\infty^2/(c_{p\infty}\,\Delta T_0)]\,(\alpha T/\bar\rho)\,\bar w\,(\partial\bar p/\partial\bar x) +$$

$$+ (4\bar\nu/3)\,[\nu_\infty/(w_\infty l)]\,[w_\infty^2/(c_{p\infty}\,\Delta T_0)]\,(\partial\bar w/\partial\bar x)^2 + [\lambda_\infty/(\rho_\infty c_{p\infty} w_\infty l)]\,(\bar\lambda/\bar\rho)\,(\partial^2\bar T/\partial\bar x^2) +$$

$$+ [\lambda_\infty/(\rho_\infty c_{p\infty}\,\nu_\infty)]\,[\nu_\infty/(w_\infty l)]\,\bar\lambda\,(\partial\bar\lambda/\partial\bar x)\,(\partial\bar T/\partial\bar x)\,. \tag{11}$$

It follows from the comparison of the equations of motion (10) for full-scale and model flows that for the hydrodynamic similarity of the flows to be ensured on condition of geometric and kinematic similarity, in addition to the equality of the known criteria Sh = $l/(w_\infty t_0)$, Eu = $p_\infty/(\rho_\infty w_\infty^2) = 1/(\mathrm{k}_\infty \mathrm{M}_\infty^2)$, Re = $w_\infty l/\nu_\infty$, Fr = $w_\infty^2/(gl)$, $\mathrm{M}_\infty = w_\infty/a_\infty$, and $\mathrm{k}_\infty = \rho_\infty a_\infty^2/p_\infty$, the Mach numbers M, indices k, and dimensionless parameters κ, ψ, and $\bar\nu$ must be the same at identical points of the flows at identical instants of time.

If in the considered flows the quantities κ and ψ are assumed to be constant (equal to their mean values), they become the determining criteria characterizing the thermodynamic properties of liquids and dense gases. For κ = const, $\mathrm{M}_{\infty\mathrm{m}} = \mathrm{M}_{\infty\mathrm{f.-sc}}$, and $\mathrm{k}_{\infty\mathrm{m}} = \mathrm{k}_{\infty\mathrm{f.-sc}}$, the indices k at identical points of similar flows of a real gas or liquid at identical instants of time will be the same. The criterion κ characterizes the fundamental properties of the medium: if $\kappa < -1$, then for $\mathrm{M} > 1$ shock waves are impossible in the liquid, but rarefaction shock waves exist [9].

It follows from the comparison of the equations of energy (11) for full-scale and model flows that for the thermal similarity of the flows to be ensured on condition of geometric, kinematic, and hydrodynamic similarity, in addition to the equality of the known criteria Pe = $w_\infty l\rho_\infty c_{p\infty}/\lambda_\infty$, Pr = $\rho_\infty c_{p\infty}\nu_\infty/\lambda_\infty$, and $\Theta = w_\infty^2/(c_{p\infty}\Delta T_0)$, the dimensionless combinations Γ_∞ and $(\alpha T)_\infty$ and the variables Γ, αT, and $\bar\lambda$ must be the same at identical points of the flows at identical instants of time. If, in the considered flows, the quantities Γ and αT can be assumed to be constant (equal to their mean values), they become the determining criteria characterizing the thermodynamic properties of liquids. The values of the relative variables $\bar\lambda$ and $\bar\nu$ or $\bar\mu = \mu/\mu_\infty = \nu\rho/(\nu_\infty\rho_\infty)$ in the model and full-scale flows will be the same if for p/p_∞ = const

$$\lambda/\lambda_\infty = (T/T_\infty)^{\tau_\lambda}\,;\quad \nu/\nu_\infty = (T/T_\infty)^{\tau_\nu}\,;\quad \mu/\mu_\infty = (T/T_\infty)^{\tau_\mu} \tag{12}$$

and for T/T_∞ = const

$$\lambda/\lambda_\infty = (p/p_\infty)^{\pi_\lambda}\,;\quad \nu/\nu_\infty = (p/p_\infty)^{\pi_\nu}\,;\quad \mu/\mu_\infty = (p/p_\infty)^{\pi_\mu}\,, \tag{13}$$

and the indices τ_λ, τ_ν, τ_μ, π_λ, π_ν, and π_μ are the same at identical points of the flows at identical instants of time. For example, $\tau_\mu = 0.76$ for air [10]; usually $\tau_\lambda = \tau_\nu$ for gases. For diesel fuels, $\pi_\nu = bp/\ln(p/p_0)$, where $b = 14\cdot10^{-3}$–$15.6\cdot10^{-6}\nu_0$. If, in the considered processes, these indices remain constant, then they will be the determining similarity criteria which characterize the variability of the coefficients of thermal conductivity and viscosity.

The criteria Γ and αT are the power expressions composed of constant parameters of different physical nature which are specified by the condition. The number of all these complex criteria can be found using the π-theorem of the dimensional theory [11]. The criteria κ, ψ, τ_λ, τ_ν, τ_μ, π_λ, π_ν, and π_μ belong to neither parametric nor complex criteria. Some of them (κ, ψ) have derivatives. Others are the exponents of parametric variables. The criteria of the third group can be obtained only from an analysis of equations describing the considered processes. Their number cannot be determined by the π-theorem of the dimensional theory.

3. In polytropic processes of compression of a real gas or a liquid with constant values of polytropic efficiency ($\eta_{\mathrm{c.pol}} = vdp/dh$) in a compressor (pump) and expansion in a turbine ($\eta_{\mathrm{exp.pol}} = dh/(vdp)$, the values

of the polytropic indices of compression n_c and expansion n_{exp} depend on the values of the adiabatic index k and the Grüneisen criterion Γ [12]:

$$n_c = k\eta_{c.pol}/[\eta_{c.pol} - \Gamma(1-\eta_{c.pol})]\,;\quad n_{exp} = k/[1+\Gamma(1-\eta_{exp.pol})]\,. \tag{14}$$

The polytropic index is the determining similarity criterion of polytropic processes. If we assume k, Γ, αT, $\eta_{c.pol}$, and $\eta_{exp.pol}$ to be constant, then for n = const we have

$$\rho_2/\rho_1 = v_1/v_2 = (p_2/p_1)^{1/n}\,;\quad T_2/T_1 = (p_2/p_1)^{[n(1+\Gamma\alpha T)-k]/(kn\alpha T)} = (p_2/p_1)^{\Gamma(\eta_{exp.pol}-1+\alpha T)/(k\alpha T)} =$$

$$= (\rho_2/\rho_1)^{\Gamma(\eta_{exp.pol}-1+\alpha T)/\left\{\alpha T[1+\Gamma(1-\eta_{exp.pol})]\right\}}\,;\quad a_2/a_1 = (p_2/p_1)^{(n-1)/(2n)}\,. \tag{15}$$

In adiabatic throttling, the polytropic efficiency $\eta_{exp.pol}$ is zero. We obtain

$$n = k/(\Gamma+1)\,;\ \rho_2/\rho_1 = v_1/v_2 = (p_2/p_1)^{(\Gamma+1)/k}\,;\ T_2/T_1 = (p_2/p_1)^{\Gamma(\alpha T-1)/(k\alpha T)}\,;\ a_2/a_1 = (p_2/p_1)^{(k-\Gamma-1)/(2k)}\,. \tag{16}$$

In these processes, $p_2 < p_1$. If $\alpha T > 1$, then $T_2 < T_1$; if $\alpha T = 1$, then $T_2 = 1$; if $\alpha T < 1$, then $T_2 > T_1$.

In liquids and dense gases with change in the pressure and for constant $\eta_{c.pol}$ and $\eta_{exp.pol}$ the polytropic index changes as sharply as the adiabatic index does, although the polytropic elasticity modulus $K_{pol} = np$ [2] remains constant or changes weakly. Similarly to κ in the isentropic process, in the polytropic process the index $\Pi = n + p(\partial n/\partial p)_\eta = (\partial K_{pol}/\partial p)_\eta$ and the parameter $B_{gyr} = p(n-\Pi)/\Pi$ remain constant or change weakly. If the values of Γ, αT, ψ, $\eta_{c.pol}$, and $\eta_{exp.pol}$ are assumed to be constant, for $\Pi = (n_2p_2 - n_1p_1)/(p_2 - p_1)$ = const and B_{gyr} = const we obtain

$$\Pi = [\kappa - \Gamma\psi(1-\eta_{exp.pol})]/[1+\Gamma(1-\eta_{exp.pol})] = [\kappa\eta_{c.pol} + \Gamma\psi(1-\eta_{c.pol})]/[\eta_{c.pol} - \Gamma(1-\eta_{c.pol})]\,;$$

$$\rho_2/\rho_1 = v_1/v_2 = [(p_2+B_{gyr})/(p_1+B_{gyr})]^{1/\Pi}\,;\quad a_2/a_1 = [(p_2+B_{gyr})/(p_1+B_{gyr})]^{(\Pi-1)/(2\Pi)-[\Gamma\psi(1-\eta_{exp.pol})]/(2\kappa)}\,;$$

$$T_2/T_1 = [(p_2+B_{gyr})/(p_1+B_{gyr})]^{[\Gamma(\eta_{exp.pol}+\alpha T-1)]/\left\{\alpha T[\kappa-\Gamma\psi(1-\eta_{exp.pol})]\right\}} =$$

$$= (\rho_2/\rho_1))]^{[\Gamma(\eta_{exp.pol}+\alpha T-1)]/\left\{\alpha T[1+\Gamma(1-\eta_{exp.pol})]\right\}}\,;$$

$$L_{c.pol} = \left\{\Pi(p_1+B_{gyr})/[(\Pi-1)\,\rho_1]\right\}\left\{[(p_2+B_{gyr})/(p_1+B_{gyr})]^{(\Pi-1)/\Pi} - 1\right\}\,; \tag{17}$$

$$L_{exp.pol} = \left\{\Pi(p_1+B_{gyr})/[(\Pi-1)\,\rho_1]\right\}\left\{1 - [(p_2+B_{gyr})/(p_1+B_{gyr})]^{(\Pi-1)/\Pi}\right\}\,.$$

In adiabatic throttling, $\Pi = (\kappa - \Gamma\psi)/(\Gamma+1)$;

$$\rho_2/\rho_1 = [(p_2+B_{gyr})/(p_1+B_{gyr})]^{1/\Pi}\,;\quad a_2/a_1 = [(p_2+B_{gyr})/(p_1+B_{gyr})]^{(\Pi-1)/(2\Pi)-\Gamma\psi/(2\kappa)}\,;$$

$$T_2/T_1 = [(p_2+B_{gyr})/(p_1+B_{gyr})]^{\Gamma(\alpha T-1)/[\alpha T(\kappa-\Gamma\psi)]} = (\rho_2/\rho_1)^{[\Gamma(\alpha T-1)]/[\alpha T(1+\Gamma)]}\,. \tag{18}$$

In the processes of change of states of liquids and dense gases, the index Π and the combination $B_{gyr}/p_{st} = (n/\Pi - 1)p/p_{st}$ are the determining similarity criteria (here p_{st} = 101,325 Pa). They also belong to the third group of criteria. The ratio B_{gyr}/p_{st} is a measure of variability of the values of k and Γ in the considered process.

Thermodynamic processes in liquids and dense gases can only approximately be calculated by the equations of polytropic processes. Formulas (8), (9), (17), and (18) describe these barotropic processes more accurately. They can be called gyrotropic [9]. They involve, as a particular case, all polytropic processes (for $B_{gyr}/p_{st} = 0$). In isentropic and isothermal processes in dense gases for $T > T_{cr.p}$ the parameters B_s and B_T are negative [13]. The suggested mathematical model of a liquid and a dense gas not only describes experimental quantities with sufficient accuracy, but also makes it possible to extrapolate by pressure to the future [14].

Example 1. To determine the pressure, density, temperature, and velocity of sound of an isentropically retarded flow of propane with M = 0.3, T = 300 K, and p = 10 MPa. From the data of [15], for T = 300 K and p = 10 MPa, ρ = 512.79 kg/m^3, s = 4.464 kJ/(kg·K), a = 847.6 m/sec, h = 596.9 kJ/kg, k = 36.84, Γ = 0.725, κ = 9.22, B_s = 29.98 MPa, and w = 254.3 m/sec. According to formulas (8), we obtain p^* = 26.92 MPa, T^* = 308.44 K, ρ^* = 532.75 kg/m^3, $h^* = h + w^2/2$ = 629.23 kJ/kg, a^* = 992.1 m/sec, and k* = 19.48. Exact calculation according to [15] gives p^* = 26.81 MPa, T^* = 308.4 K, ρ^* = 532.48 kg/m^3, a^* = 993.3 m/sec, k* = 19.6, and Γ^* = 0.736. The error of calculation by formulas (8) amounts to less than 0.4%.

In isentropic retardation of a hypothetical real gas which has the same flow parameters (p = 10 MPa, T = 300 K, ρ = 512.79 kg/m^3, a = 847.6 m/sec, M = 0.3, k = 28.16, and Γ = 0.731), we obtain p^* = 22.89 MPa, T^* = 306.3 K, ρ^* = 527.54 kg/m^3, a^* = 1263.5 m/sec, and w = 379 m/sec. It is seen that if we consider the liquid as a real gas with constant indices k and Γ (equal to the mean values of the indices of the liquid), the calculational errors increase substantially. In retardation of an ideal gas with k = 28.16 and Γ = 27.16, we obtain T^* = 666.6 K for the same values of the remaining parameters as in a real gas.

Example 2. To determine the density, temperature, and velocity of sound after adiabatic throttling of propane with p_1 = 21 MPa and T = 210 K, to p_2 = 0.5 MPa. From the data of [15], ρ_1 = 622.31 kg/m^3, k$_1$ = 59.91, $(\alpha T)_1$ = 0.320, Γ_1 = 1.4515, κ_1 = 11.37, n_1 = 24.44, k$_2$ = 1706, $(\alpha T)_2$ = 0.419, Γ_2 = 1.2487, κ_2 = 10.63, n_2 = 758.7, a_1 = 1422 m/sec, and h = 400.6 kJ/kg. According to formulas (18), Π = 6.53, B_{gyr} = 57.59 MPa, αT = 0.3695, Γ = 1.350, κ = 11.0, ρ_2 = 594.3 kg/m^3, T_2 = 219.79 K, and a_2 = 1198.1 m/sec. Exact calculation according to [15] gives ρ_2 = 594.22 kg/m^3, T_2 = 220 K, and a_2 = 1198 m/sec.

Conclusions. Criteria characterizing the thermodynamic properties of liquids and real gases are obtained. A mathematical model for calculating the processes and flows of liquids and real gases by means of the hydrodynamic functions of isentropic flows, a particular case of which is the known gasdynamic functions, is developed. It is shown that the processes in liquids and real gases cannot be described by the polytropic equations with sufficient accuracy, but they obey the equations of a wider class of barotropic processes, which can be called gyrotropic. The gyrotropic indices and some of the other obtained similarity criteria belong to neither complex nor parametric criteria. Their number cannot be determined using the π-theorem of the dimensional theory.

NOTATION

δ, specific volume; ρ, pressure; $p = 1/v$, density; s, entropy; T, temperature; K, elasticity modulus; h, enthalpy; k, adiabatic index; m, index of isotherm; c_p and c_v, isobaric and isochoric specific heat capacity, respectively; a, velocity of sound; Γ, Grüneisen criterion; $\alpha = (1/v)(\partial v/\partial T)_p$, isobaric coefficient of volumetric expansion; M, Mach number; w, velocity of flow; γ, ratio of the heat capacities; F, cross-sectional area of the flow; L, specific work; L_{eng}, engineering work done by unit mass of the liquid; L_{fr}, specific work of friction forces; Q, amount of heat taken by 1 kg of liquid from external sources; ν, kinematic viscosity; μ, dynamic viscosity; λ, thermal conductivity; g, free-fall acceleration; l, characteristic dimension of the body; t_0, characteristic time; x, distance along the abscissa; n, polytropic index; η, efficiency; Sh, Strouhal number; Eu, Euler number; Re, Reynolds number; Fr, Froude criterion; Pe, Péclet criterion; Pr, Prandtl number; Θ, temperature factor; αT, relative isobaric coefficient of volumetric expansion; κ, isentropic index; ψ, relative isobaric coefficient of change in the adiabatic elasticity modulus; B, parameter which has the dimensions of pressure; p_{st}, standard atmospheric pressure at sea level. Superscript: *, isentropically retarded parameter. Subscripts: ∞, in an undisturbed flow; w, on the body wall; c, compression; exp, expansion; pol, polytropic; cr, critical; *s*, at constant entropy; cr.p, at the thermodynamic critical point "liquid–vapor"; c.pol, in the process of compression, polytropic; exp.pol, in the process of expansion, polytropic; 0, for $p \to 0$; m, in a model flow; f.-sc, in a full-scale flow; gyr, in a gyrotropic process; *T*, at constant temperature.

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